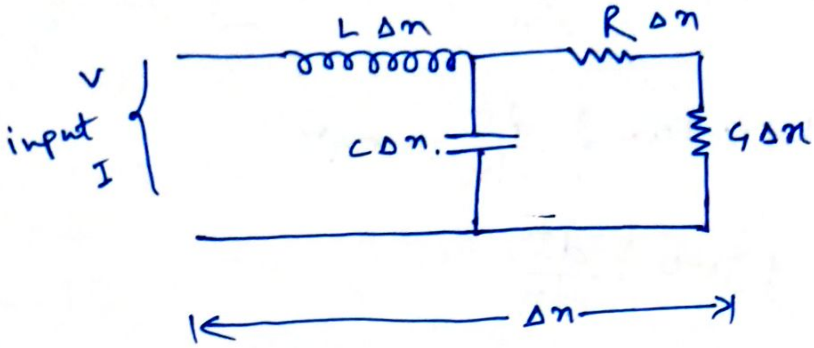
Transmission line equations & Solutions



For small length Δx

The inductance will be LΔx

capacitance " CΔx.

Resistance " RΔx.

Admittance " GΔx.

$$\Delta V = -(R\Delta x + j\omega L\Delta x).I$$

-ve sign indicates that at output there will be less value than the input value.

so

$$\lim_{\Delta x \to 0} \frac{\Delta V}{\Delta x} = -(R+j\omega L)I$$

$$\frac{dv}{dx} = -(R+j\omega L)I \quad \text{— (1)}$$

In the same manner

$$\Delta I = -(G\Delta x + j\omega C)V$$

Here -ve sign indicate that at output terminal there will be less current than the input

$$\lim_{\Delta x \to 0} \frac{\Delta I}{\Delta x} = -(G+j\omega C)V$$

or,

$$\frac{dI}{dx} = -(G+j\omega C)V \quad \text{— (2)}$$

After differentiating eq^n (1) again

$$\frac{d^2V}{dx^2} = -(R + j\omega L)\frac{dI}{dx}$$

Putting the value of $\frac{dI}{dx}$ in eq^n

$$\frac{d^2V}{dx^2} = -(R + j\omega L)\frac{dI}{dx}$$

$$\frac{d^2V}{dx^2} = -(R + j\omega L) \times -(G + j\omega C)V$$
$$= (R + j\omega L)(G + j\omega C) \cdot V.$$

$$\frac{d^2V}{dx^2} = \gamma^2 \cdot V$$

Here ~~$\gamma^2 = (R + j\omega L) \times$~~

$$\gamma^2 = (R + j\omega L)(G + j\omega C)$$

If we solve the above eq^n in space & time terms.

$$V(x,t) = V^+ e^{-\gamma x} + V^- e^{+\gamma x}.$$

for instantaneous value of voltage.

$$V(x,t) = (V^+ e^{-\gamma x} + V^- e^{+\gamma x})\, e^{j\omega t}$$

γ = Propagation constant.

Also $\gamma = \alpha + j\beta$.

α = attenuation constant.

β = phase constant.

for lossless medium

$$\alpha = 0.$$

$$V(x,t) = V^+ e^{-(\alpha + j\beta)x} \times e^{j\omega t} + V^- e^{+(\alpha + j\beta)x} e^{j\omega t}$$

$$= V^+ e^{-j\beta x} e^{j\omega t} + V^- e^{-j\beta x} e^{-j\omega t}$$

$$= V^+ e^{j(\omega t - \beta x)} + V^- e^{j(\omega t + \beta x)}.$$

$$= V^+ \cos(\omega t - \beta x) + V^- \cos(\omega t + \beta x) \quad \text{—— (A.)}$$

$\because$ $E = E_0 e^{-\alpha z} \cos(\omega t - \beta z)\hat{x}$

$E = E_0 \cos(\omega t - \beta z)\hat{x}$ (lossless medium) —— (B)

comparing the eq^n (B) & A we conclude that the voltage travelling in Transmission line like a wave.

$V^+ \cos(\omega t - \beta x) \longrightarrow$ in $+x$ direction

$V^- \cos(\omega t + \beta x) \longrightarrow$ in Backward direction.

$\because$ $\frac{dv}{dx}$